39 [L].—A. V. LUK'IANOV, I. B. TEPLOV & M. K. AKIMOVA, *Tablitsy solnovykh Kulonovskikh funktsiĭ (funktsiĭ Uittekera) Tables of Coulomb Wave Functions (Whittaker Functions)*, Moscow, 1961, xi + 223 p., 26 cm. Price 1.63 rubles.

These tables are intended for persons working in the field of nuclear physics; they were compiled in the Mathematics Section of the Physics Department of the Moscow State University in a joint project with the Laboratory of Nuclear Reactions, Scientific Research Institute for Nuclear Physics, the Moscow State University. The tabular values were computed on a STRELA computer in the University Computation Center.

The authors give the basic solutions of Coulomb functions as the functions $F_l(\eta, \rho)$ and $G_l(\eta, \rho)$, where $\eta = \frac{Zze^2}{\hbar v}$, $\rho = kr$, l is the orbital moment of the particle with only integer values, k is the wave number at infinity, ze the charge of the particle, and Ze the charge of the nucleus in whose field the particle is moving, r is the spherical coordinate, and $\hbar$ is Planck's constant divided by 2π.

Compilation of these tables was justified on the grounds that these functions are not connected with other functions by simple relationships. Available tables of Coulomb functions are essentially intended for calculating Coulomb functions at the surface of nuclei. The tables in this book are intended not only for calculating Coulomb functions on the boundary of the nuclei, but also for calculating integrals of products of radial functions encountered in the theory of direct nuclear reactions.

The tables were calculated as follows. When $\rho = 1$ the values of $F_{14}(\eta, \rho)$, $F_{15}(\eta, \rho)$, $G_0(\eta, \rho)$, $G_1(\eta, \rho)$, and their first derivatives were found by means of the series

$$F_l(\eta, \rho) = \sum_{n=l+1}^{\infty} A_n(l, \eta)\rho^n,$$

$$G_l(\eta, \rho) = F_l(\eta, \rho)K(\eta) \log \rho + \sum_{n=-l}^{\infty} B_n(l, \eta)\rho^n.$$

The coefficients $A_n(l, \eta)$, $B_n(l, \eta)$, and $K(\eta)$ can be found by formulas given in *Tables of Coulomb Wave Functions*, Applied Mathematics Series Report 17, National Bureau of Standards, 1952, and in a paper entitled "Coulomb wave functions in repulsive fields," by F. L. Yost, J. A. Wheeler, and G. Breit, in *The Physical Review*, v. 49, 1936, p. 174–189. The values of these functions for the remaining values of ρ were obtained by integrating the differential equation

$$\frac{d^2y}{d\rho^2} + \left[1 - \frac{2\eta}{\rho} - \frac{l(l+1)}{\rho^2}\right] y = 0$$

by the Runge-Kutta method programmed for automatic selection of the interval and relative accuracy of 10^{-5}, and for the remaining values of l by the recursion formula

$$\frac{1}{l+1} [\eta^2 + (l+1)^2]^{1/2} U_{l+1} + \frac{1}{l} [\eta^2 + l^2]^{1/2} U_{l-1} = (2l+1)\left[\frac{\eta}{l(l+1)} + \frac{1}{\rho}\right] U_l$$

On the basis of an error check described in the Introduction, the authors believe that the errors in the tabulated values generally do not exceed 2 to 3 units in the last decimal place, except where the function changes sign, in which case only the five tabulated decimal places are reliable. They also state that linear interpolation ensures 2 or 3 reliable decimal places, and fourth-order interpolation, 4 or 5 places. However, the accuracy of interpolation is considerably less in the region of ρ close to one, where it decreases with increasing l. In addition, 2 or 3 reliable decimal places can be obtained by linear interpolation in η, except for the region of values where $\eta > 1.5849$.

The tables for each of the functions $F_l(\eta, \rho)$, $G_l(\eta, \rho)$, and $G_l'(\eta, \rho)$ are divided into three groups corresponding to three intervals of changes in η. Part 1 (pages 2–33) gives five-place tables of values of the function $F_l(\eta, \rho)$ with $\rho = 1.0(0.2)20.0$, $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty, -0.8(0.1) - 0.4$, and $l = 0(1)15$. Part 2 (pages 36–37) gives five-place tables of $F_l(\eta, \rho)$ for the same ranges of ρ and l, $\eta = 0.50119$ (variable) 1.5849, and $\log \eta = -0.3(0.1)0.2$. Part 3 (pages 70–101) gives five-place tables of $F_l(\eta, \rho)$ for the same ranges of ρ and l, $\eta = 1.9953$ (variable) 6.3096, and $\log \eta = 0.3(0.1)0.8$.

Values of the function $G_l(\eta, \rho)$ for the same ranges of ρ and l are given as follows: on pages 104–135 for $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty, -0.8(0.1) - 0.4$; on pages 138–169 for $\eta = 0.5119$ (variable) 1.5849, $\log \eta = -0.3(0.1)0.2$; and on pages 172–203 for $\eta = 1.9953$ (variable) 6.306, $\log \eta = 0.3(0.1)0.8$. Values of the first derivative $G_l'(\eta, \rho)$ with respect to ρ, with $\rho = 1.0(0.2)20$, and $l = 0$ or 1, are given on pages 206–209, when $\eta = 0$ (variable) 0.39811, $\log \eta = -\infty, -0.8(0.1) - 0.4$; on pages 212–215, for $\eta = 0.50119$ (variable) 1.5849, $\log \eta = -0.3(0.1)0.2$; and on pages 218–221 for $\eta = 1.9953$ (variable) 6.3096, and $\log \eta = 0.3(0.1)0.8$.

References cited in the Introduction include only one Soviet source and 15 English-language sources. Among the latter are the National Bureau of Standards tables (Reports 17 and 3033), Japanese tables of Whittaker functions, and three articles by C. E. Fröberg.

The tables given in this book are arranged in a convenient manner, and the clear print adds to their attractiveness. This book should be useful to persons who who are now using the National Bureau of Standards tables.

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40 [L].—M. E. SHERRY, *The Zeros and Maxima of the Airy Function and Its First Derivative to* 25 *Significant Figures*, Electronics Research Directorate, Air Force Cambridge Research Center, Bedford, Mass., April 1959.

Table 1 gives to 25S the first 50 values of a_s and $Ai'(a_s)$ for which $Ai(a_s) = 0$. Similarly, Table 2 gives a_s' and $Ai(a_s')$ for which $Ai'(a_s') = 0$. These extend the precision of tables recorded in a rather recent volume edited by F. W. J. Olver (see *Math. Comp.*, v. 15, 1961, p. 214–215). Tables 3 and 4, respectively, give to 25S the first 18 coefficients in the asymptotic series of arc tan $\{Ai(x)/Bi(x)\}$ and arc tan $\{Ai'(x)/Bi'(x)\}$.

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